PRE-POSITIONING AND DISTRIBUTION OF EMERGENCY SUPPLY ITEMS CONSIDERING NETWORK RESILIENCE: A ROBUST OPTIMIZATION APPROACH

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ABSTRACT

Investing in preventive measures for disaster response, such as the strategic pre-positioning of emergency supplies items, is a way that humanitarian organizations can rely on to enhance their preparedness. However, it is important to consider disruptions risks when locating facility since some may become functionless. This paper presents findings obtained from a research conducted as a scientific initiation that proposes using robust optimization for emergency supply chain design that are able to meet the demand when up to Γ distribution centers were completely destroyed after a disaster strikes. The initial idea was proposed by the advisor, while the development of mathematical formulations was performed by the student under his supervision. The main activities performed by the student in this research comprised studying related literature, collaborating on model development, computational implementation, data gathering, and result analysis. Results suggest that the proposed methodology could deliver efficient disaster relief plans in real cases.

KEYWORDS. Humanitarian Logistics. Emergency Supply Distribution. Robust Optimization. Facility Location.

1 Introduction

Every year, natural and manmade disasters, such as floods, landslides, earthquakes, tornados and nuclear accidents, are reported all over the world, causing human injuries and property damage. Investing in preventive measures is a way that humanitarian organizations can enhance their emergency response capacity and preparedness before a disaster strikes (Duran et al., 2011). Preventive measures refer to actions taken before disasters occur. One element of preparedness planning can be the strategic pre-positioning of these items so that they are readily available when needed (Rawls and Turnquist, 2010).

Emergency supply items are basic elements that victims need right after a disaster occurs, such as food, vaccines, first aid items, medicines, among others. The initial time after a disaster is crucial, and the rapid and effective distribution of these items is essential to minimize casualties of those who were affected. Hence, the pre-positioning of these rescue resources is a strategy to reduce delivery time and increase preparedness, however, it requires additional investments before the emergency occurs (Bozorgi-Amiri et al., 2011).

Moreover, not taking into consideration issues related to the robustness of supplies pre-allocation plan might result in disruptions of the distribution plan, which can cause delays, or even jeopardize the post-disaster distribution. For example, the distribution center may be in a risky location that has the possibility of being destroyed or the inventory made inaccessible if a disaster occurs (Campbell and Jones, 2009). In this context, Huang et al. (2010) present a solution approach based in the fact that for large-scale disasters, such as biological attacks, hurricanes or earthquakes, most of facilities in a whole city may become functionless by the simple fact that if the area is damaged, then the facilities located too close will be damaged as well. Furthermore, Bozorgi-Amiri et al. (2011) incorporated the uncertainty not only in the supply but also in the cost and principally in the demand.

Several studies on disaster management operation propose to minimize objective functions based on the expected value of the total cost, i.e based on average, like Bozorgi-Amiri et al. (2011) and Bozorgi-Amiri et al. (2012). However, an average-value based strategy might not be suitable when dealing with humanitarian issues as this might neglect the necessity of providing service to as many people in need as possible.

In this sense, this research approaches this problem by using robust optimization under uncertainty to generate plans for pre-allocating emergency items that are robust to contingencies in order to ensure the efficient distribution of those items after the disaster strikes. The objective of this research is to develop a structured methodology to support the planning process for the distribution of emergency supply items, taking into account criteria of robustness for the generation of the emergency items allocation plan, in order to define plans that are resilient with regard to the nature of the disaster. To tackle this challenge, we propose a decision support tool based on robust optimization that is capable of defining optimal location and inventory levels of emergency supplies designing distribution networks that are able to meet the demand even if up to Γ distribution centers were completely destroyed after a disaster strikes (where Γ represents the maximum number of distribution centers that was unavailable after the event). We present a flexible methodology, in which the robustness can be traded-off with the increase in the response network cost and the distribution service level under a post-disaster situation.

The remainder of the article is organized as follows. In the next section the formulation of the problem is presented. Section 3 describes the robust optimization approach. Section 4 presents a case study and some computational experiments. Finally, Section 5 provides some relevant conclusions.

2 Problem Description

The problem considered in this study comprises two stages and four phases. The pre-disaster stage, which refers to Preventive & Mitigation and Preparedness phases, is the moment before the disaster strikes. On the other hand, the post-disaster stage, which refers to Transition/Rehabilitation and Reconstruction & Development phases, represents the stage after the disaster occurs.

In the pre-disaster stage, the pre-positioning and distribution of emergency supply items problem comprises decisions such as which facilities has to be opened and the amount of inventory level of each one considering that, after the contingency, up to Γ distribution centers might be compromised and their inventory level will be made unavailable (completely destroyed or inaccessible, for example). Taking this into account, the others selected warehouses need to have an additional quantity of emergency items to cover the absence of the damaged centers in the post-disaster phase so that all the demand is met. On the other hand, in the post-disaster stage the distribution problem determines how to distribute this emergency supply items so that they can be available to the victims. Figure I illustrates the location and inventory leveling decisions that have to be made before the disaster strikes – opening and inventory level – and the distribution decisions that have to be made after the contingency.

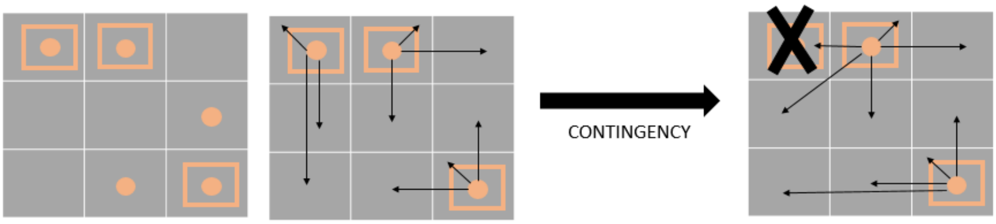

Figure I - Schematic representation of network behavior

In this paper we consider a collection of nodes that are distribution centers candidates. Each distribution center type has a maximum storage capacity ($\overline{L_{kp}}$),an annual cost of installation and operation (G_k), and a storage cost per item (S_p). We assume that we are subject to a maximum ($\overline{QD}$) and a minimum ($\underline{QD}$) number of distribution centers that can be opened. As regards to demand points, we assumed different demands at each point (D_{jp}). In this context, our decisions are which facilities to open (x_{ik}) and the average inventory level in each one (s_{ip}).

The traditional way of dealing with this type of problem is by creating an index that represents the set of all possible contingency scenarios that can occur. This scenario-based approach has been extensively used in literature to ensure that a solution obtained is feasible in all contingency scenarios considered. For instance, when we consider five chosen distribution centers and Γ equals to 2, one contingency scenario could be to consider distribution centers 1 and 3 unavailable and 2, 4 and 5 fully functional). The scenarios are denoted as c in the problem formulation.

A major drawback related with scenario-based approaches is related with the rapid increase in the number of possible contingency scenarios as the number of candidate locations and Γ increase (Street et al., 2011). Given that the size of the optimization problem is strongly connected to that quantity, it could be of great use the development of a formulation that is capable of considering all contingency scenarios without having its computational tractability jeopardized. Bearing this idea in mind, let us first state the scenario-based formulation for the pre-disaster phase network-planning problem. The formulation is stated as follows.

Parameters:

G_k	Annual cost of installation and operation of distribution center type k.
S_p	Storage Cost per unit p.
$\overline{L_{kp}}$	Maximum storage capacity in distribution center type k of item p.
$\underline{L_{kp}}$	Minimum storage capacity in distribution center type k of item p.
$\overline{QD}$	Maximum number of distribution centers to be opened.
$\underline{QD}$	Minimum number of distribution centers to be opened.
A_i^c	1 if the distribution center is available in the scenario c and 0 otherwise.
A_i	1 if the distribution center is available and 0 otherwise.
D_{jp}	Demand in demand point j of product p.

Decision Variables:

x_{ik}	1 if distribution center is open and 0 otherwise.
s_{ip}	Average inventory level at distribution center I of item p.
t_{ijp}^c	Amount of items p to transport from center i to demand point j under scenario c.
t_{ijp}	Amount of items p to transport from distribution center i to demand point j.
d^{wc*}	Maximum that can be supplied under the worst-case contingency.
$\widetilde{a_{ip}}$	1 if the distribution center is available in the worst contingency state and 0 otherwise.

$$Min \sum_{ik} G_k x_{ik} + \sum_{ip} S_p s_{ip} \tag{1}$$

$$\underline{QD} \leq \sum_{ik} x_{ik} \leq \overline{QD} \tag{2}$$

$$s_{ip} \leq \sum_{k} \overline{L_{kp}}\, x_{ik}, \forall i, p \tag{3}$$

$$s_{ip} \geq \sum_{k} \underline{L_{kp}}\, x_{ik}, \forall i, p \tag{4}$$

$$\sum_{j} t_{ijp}^{c} \leq s_{ip}\, A_{i}^{c}\,, \forall i, p, c \tag{5}$$

$$\sum_{i} t_{ijp}^{c} = D_{jp}\,, \forall j, c, p \tag{6}$$

$$\sum_{k} x_{ik} \leq 1\,, \forall i \tag{7}$$

$$x_{ik} \in \{0,1\};\ s_{ip}, t_{ijp}^{c} \geq 0. \tag{8}$$

The objective function to be minimized (1) incorporates the total cost of installing and operating warehouses, as well as the total storage cost. The limitation on the number of distribution centers to be opened is given by Constraint (2). The total inventory level of each distribution center is bounded by their maximum and minimum storage capacities, as stated by Constraints (3) and (4).

Moreover, Constraint (5) limits that the amount of items to transport from each distribution center, be less than their inventory level multiplied by the parameter A_i^c. This parameter is used to characterize contingency states in each scenario, being equal to 1 if the distribution center is available in the scenario c and 0 otherwise. In each period, $n - \Gamma$ (where n is the number of distribution centers and Γ is the maximum number of unavailable distribution centers) security criterion is enforced by considering all contingency states such that

$$\sum_{i} A_{i}^{c} \geq n - \Gamma \quad, \forall c. \tag{9}$$

Constraint (6) states that the total amount of items to be transported to each demand point must be equal to the demand. The limitation on the size of distribution centers to be opened is given by Constraint (7). Finally, (8) express the binary and non-negative nature of the variables.

In line with what was previously mentioned concerning computational tractability, one should notice that the total number of contingency scenarios that must be considered is given by $\binom{n}{\Gamma}$, which represents the total number of combinations between candidate locations (n) and unavailable locations at the post-disaster phase (Γ) that must be considered in order to ensure feasibility. Depending on how many candidates are considered and the robustness level required for the distribution network this could represent a prohibitively large number of scenarios to be considered in the optimization model.

3 Robust Optimization Approach

In this section we propose an alternative formulation for this problem that does not consider scenarios, which represents an obvious computational advantage since it reduces the complexity of the problem through the reduction of the number of variables and constraints. In order to have the model presented in a convenient way, we first propose a reformulation of such. In this sense, summing over *j* in (6) we have that

$$\sum_{ij} t_{ijp}^{c} = \sum_{j} D_{jp} \quad, \forall c, p. \tag{10}$$

Equation (5) relates the amount of items to transport from distribution center to the average inventory level. Summing over *i* in (5), the equation is equivalent to

$$\sum_{ij} t_{ijp}^{c} \leq \sum_{i} A_i^c \; s_{ip} \;, \forall c, p. \tag{11}$$

Introducing (10) in (11) yields:

$$\sum_{j} D_{jp} \leq \sum_{i} A_i^c \; s_{ip} \;, \forall c, p. \tag{12}$$

After these reformulations, the equivalent model can be stated as follows.

$$Min \sum_{ik} G_k x_{ik} + \sum_{ip} S_p s_{ip} \tag{1}$$

$$\underline{QD} \leq \sum_{ik} x_{ik} \leq \overline{QD} \tag{2}$$

$$s_{ip} \leq \sum_{k} \overline{L_{kp}} \, x_{ik}, \forall i, p \tag{3}$$

$$s_{ip} \geq \sum_{k} \underline{L_{kp}} \, x_{ik}, \forall i, p \tag{4}$$

$$\sum_{i} t_{ijp}^{c} = D_{jp} \,, \forall j, c, p \tag{6}$$

$$\sum_{k} x_{ik} \leq 1 \,, \forall i \tag{7}$$

$$\sum_{j} t_{ijp}^{c} \leq s_{ip} \;, \forall i, c, p \tag{13}$$

$$\sum_{j} D_{jp} \leq \sum_{i} A_i^c \; s_{ip} \;, \forall c, p \tag{12}$$

$$x_{ik} \in \{0,1\}; \; s_{ip}, t_{ijp}^{c} \geq 0. \tag{8}$$

The contingency state is introduced by (12). This constraint requires the total inventory level to be greater than or equal to the total demand for each contingency scenario c. Since this requirement must hold for all scenarios, it is sufficient to guarantee that it holds for the worst-case scenario, which in this case is the scenario with the tightest right-hand side of (12). Therefore, constraint (12) is equivalent to $d^{wc*} \geq \sum_j D_{jp}$, where

$$\begin{aligned} d^{wc*} = {} & Minimize_{\,\widetilde{a_{ip}}} \; \widetilde{a_{ip}} \, s_{ip} \\ & \sum_{i} \widetilde{a_{ip}} \geq n - \Gamma, \forall p \\ & 0 \leq \widetilde{a_{ip}} \leq 1 \,, \forall i, p \end{aligned}$$

The Robust Bilevel counterpart for the problem can be, thus, formulated as follows.

$$Min \sum_{ik} G_k x_{ik} + \sum_{ip} S_p s_{ip} \tag{14}$$

$$\underline{QD} \leq \sum_{ik} x_{ik} \leq \overline{QD} \tag{15}$$

$$s_{ip} \leq \sum_{k} \overline{L_{kp}} \, x_{ik}, \forall i, p \tag{16}$$

$$s_{ip} \geq \sum_{k} \underline{L_{kp}} \, x_{ik}, \forall i, p \tag{17}$$

$$\sum_{i} t_{ijp} = D_{jp}\,, \forall j, p \tag{18}$$

$$\sum_{k} x_{ik} \leq 1\,, \forall i \tag{19}$$

$$\sum_{i} t_{ijp} \leq s_{ip}\,, \forall i, p \tag{20}$$

$$d^{wc*} \geq \sum_{j} D_{jp} \tag{21}$$

$$d^{wc*} = Min_{\widetilde{a_{ip}}}\, \widetilde{a_{ip}}\, s_{ip} \tag{22}$$

$$\sum_{i} \widetilde{a_{ip}} \geq\, n - \Gamma, \forall p \tag{23}$$

$$0 \leq\, \widetilde{a_{ip}} \leq 1\,, \forall i, p \tag{24}$$

$$x_{ik} \in \{0,1\};\; s_{ip}, t_{ijp} \geq 0. \tag{25}$$

It should be noticed that the problem comprises an upper-level problem and an inner sub problem, which is not suitable for most of available commercial solver. In this sense, an equivalent single-level formulation is proposed for the bi-level problem (14) – (25). The upper-level problem imposes that the optimal objective function value of the sub problem, d^{wc*}, must be greater than or equal to the total demand, $\sum_j D_{jp}$. Bertsimas and Sim (2004) showed that by solving the dual formulation of the inner sub problem, we achieve a lower bound for d^{wc*}, based on weak duality. As this problem is convex and always has a solution, we can rely on strong duality to guarantee that d^{wc*} is always binding at the optimum. The dual formulation of the inner problem is given by

$$Max_{y_p, z_{ip}}\, (n - \Gamma) y_p - \sum_{i} z_{ip}$$

$$y_p - z_{ip} \leq s_{ip}\,, \forall i, p$$

$$z_{ip} \geq 0\,, \forall i, p$$

$$y_p \geq 0, \forall p,$$

where y and z represent the dual variables associated with constraints (23) and (24), respectively. The upper level problem consists in defining the location and allocation of the emergency supply items in the pre-contingency phase, i.e, which distribution centers should be opened and the inventory levels of each location. The inner problem's objective function represents the total post-contingency amount of items that can be transported for the demand points. Therefore, the Robust Single-level counterpart for the problem is formulated as follows.

$$Min \sum_{ik} G_k x_{ik} + \sum_{ip} S_p s_{ip} \tag{26}$$

$$\underline{QD} \leq \sum_{ik} x_{ik} \leq \overline{QD} \tag{27}$$

$$s_{ip} \leq \sum_{k} \overline{L_{kp}}\, x_{ik}, \forall i, p \tag{28}$$

$$s_{ip} \geq \sum_{k} \underline{L_{kp}}\, x_{ik}, \forall i, p \tag{29}$$

$$\sum_{i} t_{ijp} = D_{jp}\,, \forall j, p \tag{30}$$

$$\sum_{k} x_{ik} \leq 1\,, \forall i \tag{31}$$

$$\sum_{i} t_{ijp} \leq s_{ip}, \forall i,p \tag{32}$$

$$(n-\Gamma)y_p - \sum_{i} z_{ip} \geq \sum_{i} D_{jp}, \forall p \tag{33}$$

$$y_p - z_{ip} \leq s_{ip}, \forall i,p \tag{34}$$

$$z_{ip} \geq 0, \forall i,p \tag{35}$$

$$y_p \geq 0, \forall p \tag{36}$$

$$x_{ik} \in \{0,1\};\ s_{ip}, t_{ijp} \geq 0. \tag{37}$$

Expressions (14) – (20) are identical to (26) – (32). Constraint (33) correspond to (21), and (37) to (25). Finally, Constraints (34) – (36) are the dual constraints of the sub problems (22) – (24).

4 Case Study Flood in Brazil

In this section we present results from a real case study based on data available from a recent disaster occurred in Brazil. The model was implemented using the software AIMMS 3.14 in CPU Intel Core i3, 2.4-GHz processor with 4 GB. The parameters were obtained from a data base, EM-DAT (Emergency Events Database, available at www.em-dat.net/).

Table I – Product demand

Product	Demand
Food	0,20
Water	1
Hygiene	1
Cleaning	0,20
Floor	1
Medicine	0,01

Table II – Distribution Center Capacities

Capacity	Small	Medium	Large
Food	34358	385298	736238
Water	51537	577947	1104357
Hygiene	103073	1155893	2208713
Cleaning	64421	722433	1380446
Floor	10007	112223	214438
Medicine	93703	1050812	2007921

Table III – Number of victims

Nodes	Demand
TRS	16229
PTP	7214
NFB	5838
SVRP	232
BJD	2249
SMD	402
AST	771
SMM	328
SSA	107

Table IV - Size Comparison

Γ	Robust		Scenario	
	V	C	V	C
0	283	162	253	132
1	283	162	901	366
2	284	162	1333	522
3	285	162	901	366

Table V - Cost Comparison

Gama	Unavailable Facilities	Deterministic Model	Robust Model
Γ = 1	1	25.902.629,29	5.718.423,00
	2	2.672.609,47	1.656.414,00
	3	10.923.936,94	3.717.124,00
	4	2.497.760,00	1.629.070,00
Γ = 2	1 and 2	26.094.513,74	7.411.748,00
	1 and3	34.349.476,21	8.890.707,00
	1 and 4	25.902.629,27	5.876.854,00
	2 and 3	11.104.368,41	5.506.272,40
	2 and 4	2.672.609,47	1.656.414,00
	3 and 4	10.923.936,94	3.717.124,00
Average Cost		15.304.446,97	4.578.015,04

In this case study, we considered 9 demand nodes, 4 distribution centers candidates: Petrópolis, Teresópolis, Nova Friburgo and Rio de Janeiro. Moreover, we considered 6 items: food, water, hygiene, cleaning, floor, and medicine. The storage cost per unit is 21.5, 4.3, 6.7, 15.0, 7.4, and 21.3 (in R$), respectively. We also have adapted the original formulation so that 3 distinct distribution center sizes could be considered: Small, Medium and Large with 500, 800 and 1200 annual cost of installation and operation respectively. Table I represents the product demand per victim. Table II shows the capacities of each distribution center size per product. In table III we have the total number of victims in each node.

Table IV shows the number of variables (V) and constraints (C) of Scenario based and Robust model. The solutions were obtained by varying the parameter Γ that determined the robustness level. As explained in Section 3, the proposed robust model presents a more amenable scalability in regards to complexity than the scenario based model since it has fewer variables and constraints as Γ grows, that provide a significant reduction in the problem size as well as in computational requirements. Table V presents the objective function value for the deterministic model and the robust proposed model considering that one ($\Gamma = 1$) or two ($\Gamma = 2$) distribution center were unavailable after the contingency. This cost considers the costs related to the pre-disaster stage (installation and inventory costs) and to the post-disaster stage (distribution and acquisition costs). Comparing both average distribution costs, we note that the robust approach is 70,09% less than the deterministic solution. These results clearly back the superiority of the robust model over the deterministic formulation from a financial point of view.

5 Conclusions

This paper approaches the pre-positioning and distribution of emergency supply items problem. We use optimization under uncertainty to generate plans, for pre-allocating these items that are robust to contingencies in order to ensure the their efficient distribution after a disaster strikes. In this sense, we proposed a methodology based on robust optimization to define optimal location and inventory levels of those supplies guaranteeing to meet the demand even if up to Γ distribution centers were completely destroyed after the catastrophe.

Numerical results show that the proposed robust model represents a computational advantage if compared to the scenario based since not taking into account the scenarios reduces the complexity of the problem through the reduction of the number of variables and constraints. This provides a significant reduction in the problem size as well as in computational requirements. Moreover, we observe that the robust model provides a reduction of 70,09% in total cost when compared to the deterministic model.

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